

# Decays of the $B_c$ meson

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We summarize theoretical predictions on the decays of  $B_c$  meson.

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## I. INTRODUCTION

The investigation of long-lived heavy quarkonium  $B_c$  composed of quarks with different flavors can produce a significant progress in the study of heavy quark dynamics, since the variation of bound state conditions for the heavy quarks in various systems such as the heavy-light hadrons or doubly heavy mesons and baryons<sup>1</sup> provides us with the different conditions in both the binding of quarks by the strong interactions and the electroweak decays. In addition to the rich fields of study such as the spectroscopy, production mechanism and lifetime there is a possibility to get the model-independent information on the CP-violating parameters in the heavy quark sector<sup>2</sup> [2, 3].

The first experimental observation of the  $B_c$  meson by the CDF collaboration [5] confirmed the theoretical predictions on its mass, production rate and lifetime [6, 7, 8, 9, 10, 11]. So, we could expect, that an essential increase of statistics in the nearest future will provide us with a new battle field in the study of long-lived doubly heavy hadrons at Tevatron [12] and LHC [13].

Indeed, various hadronic matrix elements enter in the description of weak decays. So, measuring the lifetimes and branching ratios implies the investigation of quark confinement by the strong interactions, which is important in the evaluation of pure quark characteristics: masses and mixing angles in the CKM matrix, all of which enter as constraints on the physics beyond the Standard Model. More collection of hadrons with heavy quarks provides more accuracy and confidence in the understanding of QCD dynamics and isolation of bare quark values. So, a new lab for such investigations is a doubly heavy long-lived quarkonium  $B_c$ .

Decays of the  $B_c$  meson, containing two heavy quarks of different flavors, were considered in the pioneering paper written by Bjorken in 1986 [14]. The report was devoted to the common view onto the decays of hadrons with heavy quarks: the mesons and baryons with a single heavy quark, the  $B_c$  meson, the baryons with two and three heavy quarks. A lot of efforts was recently directed to study the long-lived doubly heavy hadrons<sup>3</sup> on the basis of modern understanding of QCD dynamics in the weak decays of heavy flavors in the framework of today approaches<sup>4</sup>: the Operator Product Expansion, sum rules of QCD [15] and NRQCD [16], and potential models adjusted due to the data on the description of hadrons with a single heavy quark. Surprisingly, the Bjorken's estimates of total widths and various branching fractions are close to what is evaluated in a more strict manner. At present we are tending to study some subtle effects caused by the influence of strong forces onto the weak decays of heavy quarks, which determines our understanding a probable fine extraction of CP-violation in the heavy quark sector.

The special role of heavy quarks in QCD is caused by the small ratio of two physical scales: the heavy quark mass  $m_Q$  is much greater than the energetic scale of the quark confinement  $\Lambda_{\text{QCD}}$ . This fact opens a way to develop two powerful tools in the description of strong interactions with the heavy quarks. The first tool is the perturbative calculations of Wilson coefficients determined by the hard corrections with the use of renormalization group improvements. The second instrument is the Operator Product Expansion (OPE) related with the small virtuality of heavy quark in the bound state, which reveals itself in many faces such as the general expansion of operators in inverse powers of heavy quark mass and the QCD sum rules. A specific form of OPE is the application for the heavy quark lagrangian itself, which results in the effective theories of heavy quarks. The effective theory is constructed under the choice of its leading term appropriate for the system under study. So, the kinetic energy can be neglected in the heavy-light

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<sup>1</sup> See the review on the physics of baryons containing two heavy quarks in [1].

<sup>2</sup> The extraction of angle  $\gamma$  in the unitary triangle derived from the charged current mixing in the heavy quark sector can be obtained from the decays of doubly heavy baryons, too, in the same manner [4].

<sup>3</sup> Reviews on the physics of  $B_c$  meson and doubly heavy baryons can be found in refs. [1, 6], respectively.

<sup>4</sup> See the program on the heavy flavour physics at Tevatron in [12].

hadrons to the leading order. The corresponding effective theory is called HQET [17]. In the doubly heavy mesons the kinetic energy is of the same order as the potential one, while the velocity of the heavy quark motion is small, and we deal with the nonrelativistic QCD [16] and its developments by taking into account the static energy in the potential NRQCD in a general form (pNRQCD) [18] or under the correlation of quarkonium size and the time for the formation of bound system with the improved scheme of expansion in the heavy quark velocity (vNRQCD) [19].

The  $B_c$  meson allows us to use such advantages like the nonrelativistic motion of  $\bar{b}$  and  $c$  quarks similar to what is known in the heavy quarkonia  $\bar{b}b$  and  $\bar{c}c$ , and suppression of light degrees of freedom: the quark-gluon sea is small in the heavy quarkonia. These two physical conditions imply two small expansion parameters for  $B_c$ :

- the relative velocity of quarks  $v$ ,
- the ratio of confinement scale to the heavy quark mass  $\Lambda_{QCD}/m_Q$ .

The perturbative QCD remains actual in the effective theories, since it is necessary for the calculation of Wilson coefficients determined by hard corrections. More definitely, the perturbative calculations determine both the matching of Wilson coefficients in the effective theory with the full QCD and the anomalous dimensions resulting in the evolution of the coefficients with the variation of normalization point. In this respect we have to mention that the pNRQCD results on the static potential and the mass-dependent terms in the heavy quark-antiquark energy were confirmed by vNRQCD after appropriate limits and some calculational corrections in vNRQCD. In addition, the pNRQCD is a powerful tool in the studies of both the spectroscopy and the heavy quarkonium decays [20].

In contrast to the Wilson coefficients, the hadronic matrix elements of operators composed by the effective fields of nonrelativistic heavy quarks cannot be evaluated in the perturbative manner. So, one should use the nonperturbative methods such as the QCD sum rules, OPE for inclusive estimates and potential models.

The measured  $B_c$  lifetime is equal to

$$\tau[B_c] = 0.46^{+0.18}_{-0.16} \pm 0.03 \text{ ps},$$

which is close to the value expected by Bjorken. The  $B_c$  decays were, at first, calculated in the PM [21, 22, 23, 24, 25, 26, 27, 28, 29, 30], wherein the variation of techniques results in close estimates after the adjustment on the semileptonic decays of  $B$  mesons. The OPE evaluation of inclusive decays gave the lifetime and widths [7], which agree with PM, if one sums up the dominating exclusive modes. That was quite unexpected, when the SR of QCD resulted in the semileptonic  $B_c$  widths [31], which are one order of magnitude less than those of PM and OPE. The reason was the valuable role of Coulomb corrections, that implies the summation of  $\alpha_s/v$  corrections significant in the heavy quarkonia, i.e. in the  $B_c$  [8, 9, 10, 23]. At present, all of mentioned approaches give the close results for the lifetime and decay modes of  $B_c$  at similar sets of parameters. Nevertheless, various dynamical questions remain open:

- What is the appropriate normalization point of non-leptonic weak lagrangian in the  $B_c$  decays, which basically determines its lifetime?
- What are the values of masses for the charmed and beauty quarks?
- What are the implications of NRQCD symmetries for the form factors of  $B_c$  decays and mode widths?
- How consistent is our understanding of hadronic matrix elements, characterizing the  $B_c$  decays, with the data on the other heavy hadrons?

In this paragraph we shortly review the  $B_c$  decays by summarizing the theoretical predictions in the different frameworks and discuss how direct experimental measurements can answer the questions above.

## II. $B_c$ LIFETIME AND INCLUSIVE DECAY RATES

The  $B_c$ -meson decay processes can be subdivided into three classes:

- 1) the  $\bar{b}$ -quark decay with the spectator  $c$ -quark,
- 2) the  $c$ -quark decay with the spectator  $\bar{b}$ -quark and
- 3) the annihilation channel  $B_c^+ \rightarrow l^+ \nu_l (c\bar{s}, u\bar{s})$ , where  $l = e, \mu, \tau$ .

In the  $\bar{b} \rightarrow \bar{c}c\bar{s}$  decays one separates also the Pauli interference with the  $c$ -quark from the initial state. In accordance with the given classification, the total width is the sum over the partial widths

$$\Gamma(B_c \rightarrow X) = \Gamma(b \rightarrow X) + \Gamma(c \rightarrow X) + \Gamma(\text{ann.}) + \Gamma(\text{PI}).$$

For the annihilation channel the  $\Gamma(\text{ann.})$  width can be reliably estimated in the framework of inclusive approach, where one takes the sum of the leptonic and quark decay modes with account for the hard gluon corrections to the effective four-quark interaction of weak currents. These corrections result in the factor of  $a_1 = 1.22 \pm 0.04$ . The width is expressed through the leptonic constant of  $f_{B_c} \approx 400$  MeV. This estimate of the quark-contribution does not depend on a hadronization model, since a large energy release of the order of the meson mass takes place. From the following expression, one can see that the contribution by light leptons and quarks can be neglected,

$$\Gamma(\text{ann.}) = \sum_{i=\tau,c} \frac{G_F^2}{8\pi} |V_{bc}|^2 f_{B_c}^2 M m_i^2 (1 - m_i^2/m_{B_c}^2)^2 \cdot C_i ,$$

where  $C_\tau = 1$  for the  $\tau^+ \nu_\tau$ -channel and  $C_c = 3|V_{cs}|^2 a_1^2$  for the  $c\bar{s}$ -channel.

As for the non-annihilation decays, in the approach of the **Operator Product Expansion** for the quark currents of weak decays [7], one takes into account the  $\alpha_s$ -corrections to the free quark decays and uses the quark-hadron duality for the final states. Then one considers the matrix element for the transition operator over the bound meson state. The latter allows one also to take into account the effects caused by the motion and virtuality of decaying quark inside the meson because of the interaction with the spectator. In this way the  $\bar{b} \rightarrow \bar{c}c\bar{s}$  decay mode turns out to be suppressed almost completely due to the Pauli interference with the charm quark from the initial state. Besides, the  $c$ -quark decays with the spectator  $\bar{b}$ -quark are essentially suppressed in comparison with the free quark decays because of a large bound energy in the initial state.

TABLE I: The branching ratios of the  $B_c$  decay modes calculated in the framework of inclusive OPE approach, by summing up the exclusive modes in the potential model [22, 23] and according to the semi-inclusive estimates in the sum rules of QCD and NRQCD [9, 10].

$B_c$ decay mode	OPE, %	PM, %	SR, %
$\bar{b} \rightarrow \bar{c}l^+\nu_l$	$3.9 \pm 1.0$	$3.7 \pm 0.9$	$2.9 \pm 0.3$
$\bar{b} \rightarrow \bar{c}u\bar{d}$	$16.2 \pm 4.1$	$16.7 \pm 4.2$	$13.1 \pm 1.3$
$\sum \bar{b} \rightarrow \bar{c}$	$25.0 \pm 6.2$	$25.0 \pm 6.2$	$19.6 \pm 1.9$
$c \rightarrow sl^+\nu_l$	$8.5 \pm 2.1$	$10.1 \pm 2.5$	$9.0 \pm 0.9$
$c \rightarrow su\bar{d}$	$47.3 \pm 11.8$	$45.4 \pm 11.4$	$54.0 \pm 5.4$
$\sum c \rightarrow s$	$64.3 \pm 16.1$	$65.6 \pm 16.4$	$72.0 \pm 7.2$
$B_c^+ \rightarrow \tau^+\nu_\tau$	$2.9 \pm 0.7$	$2.0 \pm 0.5$	$1.8 \pm 0.2$
$B_c^+ \rightarrow c\bar{s}$	$7.2 \pm 1.8$	$7.2 \pm 1.8$	$6.6 \pm 0.7$

In the framework of exclusive approach, it is necessary to sum up widths of different decay modes calculated in the potential models. While considering the semileptonic decays due to the  $\bar{b} \rightarrow \bar{c}l^+\nu_l$  and  $c \rightarrow sl^+\nu_l$  transitions, one finds that the hadronic final states are practically saturated by the lightest bound  $1S$ -state in the  $(\bar{c}c)$ -system, i.e. by the  $\eta_c$  and  $J/\psi$  particles, and the  $1S$ -states in the  $(\bar{b}s)$ -system, i.e.  $B_s$  and  $B_s^*$ , which can only enter the accessible energetic gap.

Further, the  $\bar{b} \rightarrow \bar{c}u\bar{d}$  channel, for example, can be calculated through the given decay width of  $\bar{b} \rightarrow \bar{c}l^+\nu_l$  with account for the color factor and hard gluon corrections to the four-quark interaction. It can be also obtained as a sum over the widths of decays with the  $(u\bar{d})$ -system bound states.

The results of calculation for the total  $B_c$  width in the inclusive OPE and exclusive PM approaches give the values consistent with each other, if one takes into account the most significant uncertainty related to the choice of quark masses (especially for the charm quark), so that finally, we have

$$\tau[B_c^+]_{\text{OPE, PM}} = 0.55 \pm 0.15 \text{ ps}, \quad (1)$$

which agrees with the measured value of  $B_c$  lifetime.

The OPE estimates of inclusive decay rates agree with recent semi-inclusive calculations in the sum rules of QCD and NRQCD [9, 10], where one assumed the saturation of hadronic final states by the ground levels in the  $c\bar{c}$  and  $\bar{b}s$  systems as well as the factorization allowing one to relate the semileptonic and hadronic decay modes. The coulomb-like corrections in the heavy quarkonia states play an essential role in the  $B_c$  decays and allow one to remove the disagreement between the estimates in sum rules and OPE. In contrast to OPE, where the basic uncertainty is given by the variation of heavy quark masses, these parameters are fixed by the two-point sum rules for bottomonia and charmonia, so that the accuracy of SR calculations for the total width of  $B_c$  is determined by the choice of scale  $\mu$  for

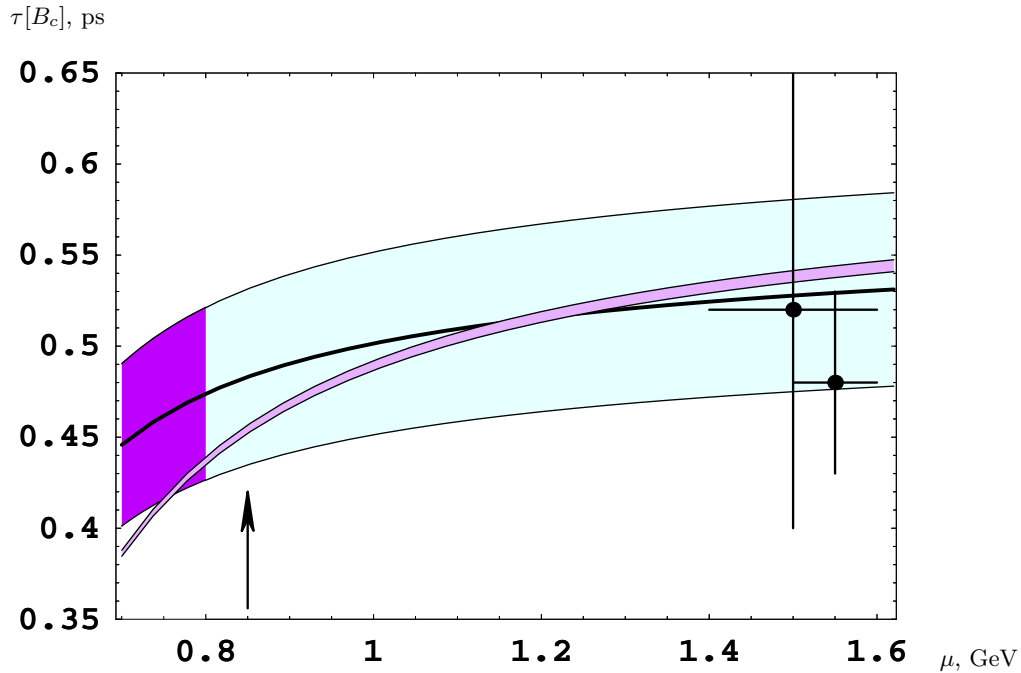


FIG. 1: The  $B_c$  lifetime calculated in QCD sum rules versus the scale of hadronic weak lagrangian in the decays of charmed quark. The wide shaded region shows the uncertainty of semi-inclusive estimates, the dark shaded region is the preferable choice as given by the lifetimes of charmed mesons. The dots represent the values in OPE approach taken from ref. [7]. The narrow shaded region represents the result obtained by summing up the exclusive channels with the variation of hadronic scale in the decays of beauty anti-quark in the range of  $1 < \mu_b < 5$  GeV. The arrow points to the preferable prescription of  $\mu = 0.85$  GeV as discussed in [10].

the hadronic weak lagrangian in decays of charmed quark. We show this dependence in Fig. 1, where  $\frac{m_c}{2} < \mu < m_c$  and the dark shaded region corresponds to the scales preferred by data on the charmed meson lifetimes.

Supposing the preferable choice of scale in the  $c \rightarrow s$  decays of  $B_c$  to be equal to  $\mu_{B_c}^2 \approx (0.85 \text{ GeV})^2$ , putting  $a_1(\mu_{B_c}) = 1.20$  and neglecting the contributions caused by nonzero  $a_2$  in the charmed quark decays [10], in the framework of semi-inclusive sum-rule calculations we predict

$$\tau[B_c]_{\text{SR}} = 0.48 \pm 0.05 \text{ ps}, \quad (2)$$

which agrees with the direct summation of exclusive channels calculated in the next sections. In Fig. 1 we show the exclusive estimate of lifetime, too [11].

### III. EXCLUSIVE DECAYS

Predictions for the exclusive decays of  $B_c$  are summarized in Table II at the fixed values of factors  $a_{1,2}$  (see below) and lifetime. We put the predicted values close to those of QCD sum rules, which have an uncertainty about 15% and basically agree with the most of potential models, while marginal deviations expected in some potential models are shown in square brackets.

In addition to the decay channels with the heavy charmonium  $J/\psi$  well detectable through its leptonic mode, one could expect a significant information on the dynamics of  $B_c$  decays from the channels with a single heavy mesons, if an experimental efficiency allows one to extract a signal from the cascade decays. An interesting opportunity is presented by the relations for the ratios in (28), which can shed light to characteristics of the non-leptonic decays in the explicit form.

We have found that the  $\bar{b}$  decay to the doubly charmed states gives

$$\text{Br}[B_c^+ \rightarrow \bar{c}c c\bar{s}] \approx 1.39\%,$$

TABLE II: Branching ratios of exclusive  $B_c^+$  decays at the fixed choice of factors:  $a_1^c = 1.20$  and  $a_2^c = -0.317$  in the non-leptonic decays of  $c$  quark, and  $a_1^b = 1.14$  and  $a_2^b = -0.20$  in the non-leptonic decays of  $\bar{b}$  quark. The lifetime of  $B_c$  is appropriately normalized by  $\tau[B_c] \approx 0.45$  ps. The numbers in square brackets present the marginal values obtained in some potential models in order to show possible range of variation.

Mode	BR, %	Mode	BR, %	Mode	BR, %
$B_c^+ \rightarrow \eta_c e^+ \nu$	0.75 [0.5]	$B_c^+ \rightarrow J/\psi K^+$	0.011 [0.007]	$B_c^+ \rightarrow B_s^0 K^+$	1.06 [0.2]
$B_c^+ \rightarrow \eta_c \tau^+ \nu$	0.23 [0.2]	$B_c \rightarrow J/\psi K^{*+}$	0.022 [0.016]	$B_c^+ \rightarrow B_s^{*0} K^+$	0.37 [0.13]
$B_c^+ \rightarrow \eta_c' e^+ \nu$	0.020 [0.05]	$B_c^+ \rightarrow D^+ \bar{D}^0$	0.0053 [0.0018]	$B_c^+ \rightarrow B_s^0 K^{*+}$	–
$B_c^+ \rightarrow \eta_c' \tau^+ \nu$	0.0016 [–]	$B_c^+ \rightarrow D^+ \bar{D}^{*0}$	0.0075 [0.002]	$B_c^+ \rightarrow B_s^{*0} K^{*+}$	–
$B_c^+ \rightarrow J/\psi e^+ \nu$	1.9 [1]	$B_c^+ \rightarrow D^{*+} \bar{D}^0$	0.0049 [0.0009]	$B_c^+ \rightarrow B^0 \pi^+$	1.06 [0.1]
$B_c^+ \rightarrow J/\psi \tau^+ \nu$	0.48 [0.35]	$B_c^+ \rightarrow D^{*+} \bar{D}^{*0}$	0.033 [0.003]	$B_c^+ \rightarrow B^0 \rho^+$	0.96 [0.2]
$B_c^+ \rightarrow \psi' e^+ \nu$	0.094 [0.2]	$B_c^+ \rightarrow D_s^+ \bar{D}^0$	0.00048 [0.0001]	$B_c^+ \rightarrow B^{*0} \pi^+$	0.95 [0.08]
$B_c^+ \rightarrow \psi' \tau^+ \nu$	0.008 [–]	$B_c^+ \rightarrow D_s^+ \bar{D}^{*0}$	0.00071 [0.00012]	$B_c^+ \rightarrow B^{*0} \rho^+$	2.57 [0.6]
$B_c^+ \rightarrow D^0 e^+ \nu$	0.004 [0.02]	$B_c^+ \rightarrow D_s^{*+} \bar{D}^0$	0.00045 [0.00005]	$B_c^+ \rightarrow B^0 K^+$	0.07 [0.01]
$B_c^+ \rightarrow D^0 \tau^+ \nu$	0.002 [0.08]	$B_c^+ \rightarrow D_s^{*+} \bar{D}^{*0}$	0.0026 [0.0002]	$B_c^+ \rightarrow B^0 K^{*+}$	0.015 [0.012]
$B_c^+ \rightarrow D^{*0} e^+ \nu$	0.018 [0.004]	$B_c^+ \rightarrow \eta_c D_s^+$	0.28 [0.07]	$B_c^+ \rightarrow B^{*0} K^+$	0.055 [0.006]
$B_c^+ \rightarrow D^{*0} \tau^+ \nu$	0.008 [0.016]	$B_c^+ \rightarrow \eta_c D_s^{*+}$	0.27 [0.07]	$B_c^+ \rightarrow B^{*0} K^{*+}$	0.058 [0.04]
$B_c^+ \rightarrow B_s^0 e^+ \nu$	4.03 [1]	$B_c^+ \rightarrow J/\psi D_s^+$	0.17 [0.05]	$B_c^+ \rightarrow B^+ \bar{K}^0$	1.98 [0.18]
$B_c^+ \rightarrow B_s^{*0} e^+ \nu$	5.06 [1.2]	$B_c^+ \rightarrow J/\psi D_s^{*+}$	0.67 [0.5]	$B_c^+ \rightarrow B^+ \bar{K}^{*0}$	0.43 [0.09]
$B_c^+ \rightarrow B^0 e^+ \nu$	0.34 [0.08]	$B_c^+ \rightarrow \eta_c D^+$	0.015 [0.04]	$B_c^+ \rightarrow B^{*+} \bar{K}^0$	1.60 [0.06]
$B_c^+ \rightarrow B^{*0} e^+ \nu$	0.58 [0.15]	$B_c^+ \rightarrow \eta_c D^{*+}$	0.010 [0.002]	$B_c^+ \rightarrow B^{*+} \bar{K}^{*0}$	1.67 [0.6]
$B_c^+ \rightarrow \eta_c \pi^+$	0.20 [0.12]	$B_c^+ \rightarrow J/\psi D^+$	0.009 [0.002]	$B_c^+ \rightarrow B^+ \pi^0$	0.037 [0.004]
$B_c^+ \rightarrow \eta_c \rho^+$	0.42 [0.3]	$B_c^+ \rightarrow J/\psi D^{*+}$	0.028 [0.014]	$B_c^+ \rightarrow B^+ \rho^0$	0.034 [0.01]
$B_c^+ \rightarrow J/\psi \pi^+$	0.13 [0.08]	$B_c^+ \rightarrow B_s^0 \pi^+$	16.4 [1.6]	$B_c^+ \rightarrow B^{*+} \pi^0$	0.033 [0.003]
$B_c^+ \rightarrow J/\psi \rho^+$	0.40 [0.2]	$B_c^+ \rightarrow B_s^0 \rho^+$	7.2 [2.4]	$B_c^+ \rightarrow B^{*+} \rho^0$	0.09 [0.03]
$B_c^+ \rightarrow \eta_c K^+$	0.013 [0.008]	$B_c^+ \rightarrow B_s^{*0} \pi^+$	6.5 [1.3]	$B_c^+ \rightarrow \tau^+ \nu_\tau$	1.6
$B_c^+ \rightarrow \eta_c K^{*+}$	0.020 [0.018]	$B_c^+ \rightarrow B_s^{*0} \rho^+$	20.2 [11]	$B_c^+ \rightarrow c \bar{s}$	4.9

so that in the absolute value of width it can be compared with the estimate of spectator decay [7],

$$\Gamma[B_c^+ \rightarrow \bar{c} c c \bar{s}]|_{\text{SR}} \approx 20 \cdot 10^{-15} \text{ GeV},$$

$$\Gamma[B_c^+ \rightarrow \bar{c} c c \bar{s}]|_{\text{spect.}} \approx 90 \cdot 10^{-15} \text{ GeV},$$

and we find the suppression factor of about 1/4.5. This result is in agreement with the estimate in OPE [7], where a strong dependence of negative term caused by the Pauli interference on the normalization scale of non-leptonic weak lagrangian was emphasized, so that at moderate scales one gets approximately the same suppression factor, too.

To the moment we certainly state that the accurate direct measurement of  $B_c$  lifetime can provide us with the information on both the masses of charmed and beauty quarks and the normalization point of non-leptonic weak lagrangian in the  $B_c$  decays (the  $a_1$  and  $a_2$  factors). The experimental study of semileptonic decays and the extraction of ratios for the form factors can test the spin symmetry derived in the NRQCD and HQET approaches and decrease the theoretical uncertainties in the corresponding theoretical evaluation of quark parameters as well as the hadronic matrix elements, determined by the nonperturbative effects caused by the quark confinement. The measurement of branching fractions for the semileptonic and non-leptonic modes and their ratios can inform on the values of factorization parameters, which depend again on the normalization of non-leptonic weak lagrangian. The charmed quark counting in the  $B_c$  decays is related to the overall contribution of  $b$  quark decays as well as with the suppression of  $\bar{b} \rightarrow c \bar{c} \bar{s}$  transition because of the destructive interference, which value depends on the nonperturbative parameters (roughly estimated, the leptonic constant) and non-leptonic weak lagrangian.

Thus, the progress in measuring the  $B_c$  lifetime and decays could enforce the theoretical understanding of what really happens in the heavy quark decays at all.

### A. Semileptonic decays

The semileptonic decay rates estimated in the QCD sum rules for 3-point correlators [32] are underestimated in [31], because large coulomb-like corrections were not taken into account. The recent analysis of SR in [8, 9, 10] decreased the uncertainty, so that the estimates agree with the calculations in the potential models.

#### 1. Coulomb corrections in the heavy quarkonia

For the heavy quarkonium  $\bar{b}c$ , where the relative velocity of quark movement is small, an essential role is taken by the Coulomb-like  $\alpha_s/v$ -corrections. They are caused by the ladder diagram, shown in Fig. 2. It is well known that an account for this corrections in two-point sum rules numerically leads to a double-triple multiplication of Born value of spectral density [15]. In our case it leads to the finite renormalization for  $\rho_i$  [9], so that

$$\rho_i^c = \mathcal{C}\rho_i, \quad (3)$$

with

$$\mathcal{C}^2 = \left| \frac{\Psi_{\bar{b}c}^c(0)}{\Psi_{\bar{b}c}^{\text{free}}(0)} \right|^2 = \frac{4\pi\alpha_s^c}{3v} \frac{1}{1 - \exp\left(-\frac{4\pi\alpha_s^c}{3v}\right)}, \quad (4)$$

where  $v$  is the relative velocity of quarks in the  $\bar{b}c$ -system,

$$v = \sqrt{1 - \frac{4m_b m_c}{p_1^2 - (m_b - m_c)^2}}, \quad (5)$$

and the coupling constant of effective coulomb interactions  $\alpha_s^c$  should be prescribed by the calculations of leptonic constants for the appropriate heavy quarkonia as described in the next section.

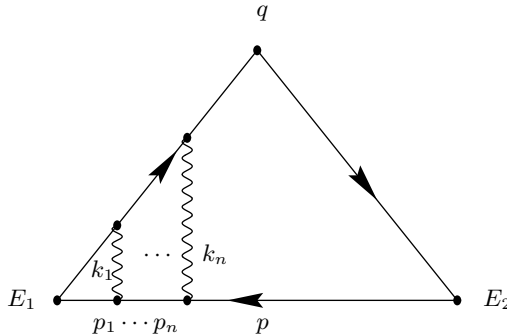


FIG. 2: The ladder diagram of the Coulomb-like interaction.

A similar coulomb factor appears in the vertex of heavy quarks composing the final heavy quarkonium, in the case of  $\bar{c}c$ .

In order to fix such the parameters as the heavy quark masses and effective couplings of coulomb exchange in the nonrelativistic systems of heavy quarkonia with the same accuracy used in the three-point sum rules, we explore the two-point sum rules of QCD for the systems of  $\bar{c}c$ ,  $\bar{b}c$  and  $\bar{b}b$ . Thus, we take into account the quark loop contribution with the coulomb factors like that of (4). We keep this procedure despite of current status of NRQCD sum rules for the heavy quarkonium, wherein the three-loop corrections to the correlators are available to the moment (see [33] for review), since for the sake of consistency, the calculations should be performed in the same order for both the three-point and two-point correlators. This fact follows from the expression for the resonance term in the three-point correlator, so that the form factor of transition involves the normalization by the leptonic constants extracted from the two-point sum rules. This procedure is taken since calculations of two-loop corrections to the three-point correlators are not available to the moment, unfortunately.

Then, the use of experimental values for the leptonic constants of charmonium and bottomonium in addition to the consistent description of spectral function moments in the two-point sum rules allows us to extract the effective couplings of coulomb exchange as well as the heavy quark masses in the heavy quarkonium channels. We have found that the normalization of leptonic constant is fixed by the appropriate choice of effective constant for the coulomb exchange  $\alpha_s^C$ , while the stability is very sensitive to the prescribed value of heavy quark mass. The physical meaning of such heavy quark masses is determined by the threshold posing the energy at which the coulomb spectrum starts, so that it is very close to the so-called ‘potential subtracted masses’ of heavy quarks,  $m^{\text{PS}}$  known in the literature (see [34]) in the context of renormalon in the perturbative pole mass [35]. We have found that the numerical values obtained coincide with the appropriate  $m^{\text{PS}}$  within the error bars.

## 2. Primary modes

In practice, the most constructive information is given by the  $\psi$  mode, since this charmonium is clearly detected in experiments due to the pure leptonic decays [5]. In addition to the investigation of various form factors and their dependence on the transfer squared, we would like to stress that the measurement of decay to the excited state of charmonium, i.e.  $\psi'$ , could answer the question on the reliability of QCD predictions for the decays to the excited states. We see that to the moment the finite energy sum rules predict the width of  $B_c^+ \rightarrow \psi' l^+ \nu$  decays in a reasonable agreement with the potential models if one takes into account an uncertainty about 50%.

## 3. Relations between the form factors

In the limit of infinitely heavy quark mass, the NRQCD and HQET lagrangians possess the spin symmetry, since the heavy quark spin is decoupled in the leading approximation. The most familiar implication of such the symmetry is the common Isgur-Wise function determining the form factors in the semileptonic decays of singly heavy hadrons.

In contrast to the weak decays with the light spectator quark, the  $B_c$  decays to both the charmonia  $\psi$  and  $\eta_c$  and  $B_s^{(*)}$  involve the heavy spectator, so that the spin symmetry works only at the recoil momenta close to zero, where the spectator enters the heavy hadron in the final state with no hard gluon rescattering. Hence, in a strict consideration we expect the relations between the form factors in the vicinity of zero recoil. The normalization of common form factor is not fixed, as was in decays of hadrons with a single heavy quark, since the heavy quarkonia wave-functions are flavour-dependent. Nevertheless, in practice, the ratios of form factors as fixed at a given zero recoil point are broken only by the different dependence on the transfer squared, that is not significant in real numerical estimates in the restricted region of physical phase space.

As for the implications of spin symmetry for the form factors of decay, in the soft limit for the transitions  $B_c^+ \rightarrow \psi(\eta_c) e^+ \nu$

$$\begin{aligned} v_1^\mu &\neq v_2^\mu, \\ w &= v_1 \cdot v_2 \rightarrow 1, \end{aligned} \tag{6}$$

where  $v_{1,2}^\mu = p_{1,2}^\mu / \sqrt{p_{1,2}^2}$  are the four-velocities of heavy quarkonia in the initial and final states, we derive the relations [9]

$$f_+(c_1^P \cdot \mathcal{M}_2 - c_2^P \cdot \mathcal{M}_1) - f_-(c_1^P \cdot \mathcal{M}_2 + c_2^P \cdot \mathcal{M}_1) = 0, \quad F_0^A \cdot c_V - 2c_\epsilon \cdot F_V \mathcal{M}_1 \mathcal{M}_2 = 0, \tag{7}$$

$$F_0^A(c_1 + c_2) - c_\epsilon \mathcal{M}_1 (F_+^A(\mathcal{M}_1 + \mathcal{M}_2) + F_-^A(\mathcal{M}_1 - \mathcal{M}_2)) = 0, \quad F_0^A c_1^P + c_\epsilon \cdot \mathcal{M}_1 (f_+ + f_-) = 0, \tag{8}$$

where

$$\begin{aligned} c_\epsilon &= -2, & c_1 &= -\frac{m_3(3m_1 + m_3)}{4m_1 m_2}, & c_2 &= \frac{1}{4m_1 m_2} (4m_1 m_2 + m_1 m_3 + 2m_2 m_3 + m_3^2), \\ c_1^P &= 1 + \frac{m_3}{2m_1} - \frac{m_3}{2m_2}, & c_2^P &= 1 - \frac{m_3}{2m_1} + \frac{m_3}{2m_2}, & c_V &= -\frac{1}{2m_1 m_2} (2m_1 m_2 + m_1 m_3 + m_2 m_3), \end{aligned} \tag{9}$$

so that  $m_1$  is the mass of decaying quark,  $m_2$  is the quark mass of decay product, and  $m_3$  is the mass of spectator quark, while  $\mathcal{M}_1 = m_1 + m_2$ ,  $\mathcal{M}_2 = m_2 + m_3$ .

The SR estimates of form factors show a good agreement with the relations, whereas the deviations can be basically caused by the difference in the  $q^2$ -evolution of form factors from the zero recoil point, that can be neglected within the accuracy of SR method for the transitions of  $B_c \rightarrow \bar{c}c$  as shown in [9].

In the same limit for the semileptonic modes with a single heavy quark in the final state we find that the ambiguity in the ‘light quark propagator’ (strictly, we deal with the uncertainty in the spin structure of amplitude because of light degrees of freedom) restricts the number of relations, and we derive

$$f_+(\bar{c}_1^P \cdot \mathcal{M}_2 - \bar{c}_2^P \cdot \mathcal{M}_1) - f_-(\bar{c}_1^P \cdot \mathcal{M}_2 + \bar{c}_2^P \cdot \mathcal{M}_1) = 0, \quad F_0^A \cdot \bar{c}_V - 2\bar{c}_\epsilon \cdot F_V \mathcal{M}_1 \mathcal{M}_2 = 0, \quad F_0^A \bar{c}_1^P + \bar{c}_\epsilon \cdot \mathcal{M}_1 (f_+ + f_-) = 0, \quad (10)$$

where

$$\bar{c}_\epsilon = -2, \quad \bar{c}_V = -1 - \tilde{B} - \frac{m_3}{2m_1}, \quad \bar{c}_1^P = 1 - \tilde{B} + \frac{m_3}{2m_1}, \quad \bar{c}_2^P = 1 + \tilde{B} - \frac{m_3}{2m_1}, \quad (11)$$

so that  $m_2$  is the mass of the light quark. The parameter  $\tilde{B}$  has the form

$$\tilde{B} = -\frac{2m_1 + m_3}{2m_1} + \frac{4m_3(m_1 + m_3)F_V}{F_0^A}. \quad (12)$$

The  $1/m_Q$ -deviations from the symmetry relations in the decays of  $B_c^+ \rightarrow B_s^{(*)} e^+ \nu$  are about 10-15 %, as found in the QCD sum rules considered in [10].

Next, we investigate the validity of spin-symmetry relations in the  $B_c$  decays to  $B^{(*)}$ ,  $D^{(*)}$  and  $D_s^{(*)}$ . The results of estimates for the  $f_\pm$  evaluated by the symmetry relations with the inputs given by the form factors  $F_V$  and  $F_0^A$  extracted from the sum rules have been compared with the values calculated in the framework of sum rules.

We have found that the uncertainty in the estimates is basically determined by the variation of pole masses in the  $q^2$ -dependencies of form factors, which govern the evolution from the zero recoil point to the zero transfer squared. So, the variation of  $M_{\text{pole}}[f_\pm]$  in the range of 4.8 – 5 GeV for the transitions of  $B_c \rightarrow D^{(*)}$  and  $B_c \rightarrow D_s^{(*)}$  results in the 30%-uncertainty in the form factors. Analogously, the variation of  $M_{\text{pole}}[f_\pm]$  in the range of 1.5 – 1.9 GeV for the transition of  $B_c \rightarrow B^{(*)}$  results in the uncertainty about 35%.

Note, that the combinations of relations given above reproduce the only equality [36], which was found for each mode in the strict limit of  $v_1 = v_2$ .

## B. Leptonic decays

The dominant leptonic decay of  $B_c$  is given by the  $\tau\nu_\tau$  mode (see Table I). However, it has a low experimental efficiency of detection because of hadronic background in the  $\tau$  decays or a missing energy. Recently, in refs. [37] the enhancement of muon and electron channels in the radiative modes was studied. The additional photon allows one to remove the helicity suppression for the leptonic decay of pseudoscalar particle, which leads, say, to the double increase of muonic mode.

### 1. Leptonic constant of $B_c$

In the NRQCD approximation for the heavy quarks, the calculation of leptonic constant for the heavy quarkonium with the two-loop accuracy requires the matching of NRQCD currents with the currents in full QCD,

$$J_\nu^{\text{QCD}} = \bar{Q}_1 \gamma_5 \gamma_\nu Q_2, \quad \mathcal{J}_\nu^{\text{NRQCD}} = -\chi^\dagger \phi v_\nu,$$

where we have introduced the following notations:  $Q_{1,2}$  are the relativistic quark fields,  $\chi$  and  $\phi$  are the nonrelativistic spinors of anti-quark and quark,  $v$  is the four-velocity of heavy quarkonium, so that

$$J_\nu^{\text{QCD}} = \mathcal{K}(\mu_{\text{hard}}; \mu_{\text{fact}}) \cdot \mathcal{J}_\nu^{\text{NRQCD}}(\mu_{\text{fact}}), \quad (13)$$

where the scale  $\mu_{\text{hard}}$  gives the normalization point for the matching of NRQCD with full QCD, while  $\mu_{\text{fact}}$  denotes the normalization point for the calculations in the perturbation theory of NRQCD.

For the pseudoscalar heavy quarkonium composed of heavy quarks with the different flavors, the Wilson coefficient  $\mathcal{K}$  is calculated with the two-loop accuracy

$$\mathcal{K}(\mu_{\text{hard}}; \mu_{\text{fact}}) = 1 + c_1 \frac{\alpha_s^{\overline{\text{MS}}}(\mu_{\text{hard}})}{\pi} + c_2(\mu_{\text{hard}}; \mu_{\text{fact}}) \left( \frac{\alpha_s^{\overline{\text{MS}}}(\mu_{\text{hard}})}{\pi} \right)^2, \quad (14)$$



and  $c_{1,2}$  are explicitly given in Refs. [38] and [39], respectively. The anomalous dimension of factor  $\mathcal{K}(\mu_{\text{hard}}; \mu_{\text{fact}})$  in NRQCD is defined by

$$\frac{d \ln \mathcal{K}(\mu_{\text{hard}}; \mu)}{d \ln \mu} = \sum_{k=1}^{\infty} \gamma_{[k]} \left( \frac{\alpha_s^{\overline{\text{MS}}}(\mu)}{4\pi} \right)^k, \quad (15)$$

whereas the two-loop calculations<sup>5</sup> give

$$\gamma_{[1]} = 0, \quad (16)$$

$$\gamma_{[2]} = -8\pi^2 C_F \left[ \left( 2 - \frac{(1-r)^2}{(1+r)^2} \right) C_F + C_A \right], \quad (17)$$

where  $r$  denotes the ratio of heavy quark masses. The initial condition for the evolution of factor  $\mathcal{K}(\mu_{\text{hard}}; \mu_{\text{fact}})$  is given by the matching of NRQCD current with full QCD at  $\mu_{\text{fact}} = \mu_{\text{hard}}$ .

The leptonic constant is defined in the following way:

$$\langle 0 | J_{\nu}^{\text{QCD}} | \bar{Q}Q \rangle = v_{\nu} f_{\bar{Q}Q} M_{\bar{Q}Q}. \quad (18)$$

In full QCD the axial vector current of quarks has zero anomalous dimension, while in NRQCD the current  $\mathcal{J}_{\nu}^{\text{NRQCD}}$  has the nonzero anomalous dimension, so that in accordance with (13)–(17), we find

$$\langle 0 | \mathcal{J}_{\nu}^{\text{NRQCD}}(\mu) | \bar{Q}Q \rangle = \mathcal{A}(\mu) v_{\nu} f_{\bar{Q}Q}^{\text{NRQCD}} M_{\bar{Q}Q}, \quad (19)$$

where, in terms of nonrelativistic quarks, the leptonic constant for the heavy quarkonium is given by the well-known relation with the wave function at the origin

$$f_{\bar{Q}Q}^{\text{NRQCD}} = \sqrt{\frac{12}{M_{\bar{Q}Q}}} |\Psi_{\bar{Q}Q}(0)|, \quad (20)$$

and the value of wave function in the leading order is determined by the solution of Schrödinger equation with the static potential, so that we isolate the scale dependence of NRQCD current in the factor  $\mathcal{A}(\mu)$ , while the leptonic constant  $f_{\bar{Q}Q}^{\text{NRQCD}}$  is evaluated at a fixed normalization point  $\mu = \mu_0$ , which will be attributed below. It is evident that

$$f_{\bar{Q}Q} = f_{\bar{Q}Q}^{\text{NRQCD}} \mathcal{A}(\mu_{\text{fact}}) \cdot \mathcal{K}(\mu_{\text{hard}}; \mu_{\text{fact}}), \quad (21)$$

and the anomalous dimension of  $\mathcal{A}(\mu_{\text{fact}})$  should compensate the anomalous dimension of factor  $\mathcal{K}(\mu_{\text{hard}}; \mu_{\text{fact}})$ , so that in two loops we have got

$$\frac{d \ln \mathcal{A}(\mu)}{d \ln \mu} = -\gamma_{[2]} \left( \frac{\alpha_s^{\overline{\text{MS}}}(\mu)}{4\pi} \right)^2. \quad (22)$$

The physical meaning of  $\mathcal{A}(\mu)$  is clearly determined by the relations of (19) and (21): this factor gives the normalization of matrix element for the current of nonrelativistic quarks expressed in terms of wave function for the two-particle quark state (in the leading order of inverse heavy quark mass in NRQCD). Certainly, in this approach the current of nonrelativistic quarks is factorized from the quark-gluon sea, which is a necessary attribute of hadronic state, so that, in general, this physical state can be only approximately represented as the two-quark bound state. In the consideration of leptonic constants in the framework of NRQCD, this approximation requires to introduce the normalization factor  $\mathcal{A}(\mu)$  depending on the scale.

The renormalization group equation of (22) is simply integrated out, so that

$$\mathcal{A}(\mu) = \mathcal{A}(\mu_0) \left[ \frac{\beta_0 + \beta_1 \frac{\alpha_s^{\overline{\text{MS}}}(\mu)}{4\pi}}{\beta_0 + \beta_1 \frac{\alpha_s^{\overline{\text{MS}}}(\mu_0)}{4\pi}} \right]^{\frac{\gamma_{[2]}}{2\beta_1}}, \quad (23)$$

---

<sup>5</sup> We use ordinary notations for the invariants of  $SU(N_c)$  representations:  $C_F = \frac{N_c^2 - 1}{2N_c}$ ,  $C_A = N_c$ ,  $T_F = \frac{1}{2}$ ,  $n_f$  is a number of “active” light quark flavors.

where  $\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$ , and  $\beta_1 = \frac{34}{3}C_A^2 - 4C_F T_F n_f - \frac{20}{3}C_A T_F n_f$ . A constant of integration could be defined so that at a scale  $\mu_0$  we would get  $\mathcal{A}(\mu_0) = 1$ . Thus, in the framework of NRQCD we have got the parametric dependence of leptonic constant estimates on the scale  $\mu_0$ , which has the following simple interpretation: the normalization of matrix element for the current of nonrelativistic quarks at  $\mu_0$  is completely given by the wave function of two-quark bound state. At other  $\mu \neq \mu_0$  we have to introduce the factor  $\mathcal{A}(\mu) \neq 1$ , so that the approximation of hadronic state by the two-quark wavefunction becomes inexact.

Following the method described in [40, 41], we estimate the wave function of  $\bar{b}c$  quarkonium in the nonrelativistic model with the static potential given by [40]. Details of calculations are presented in [42].

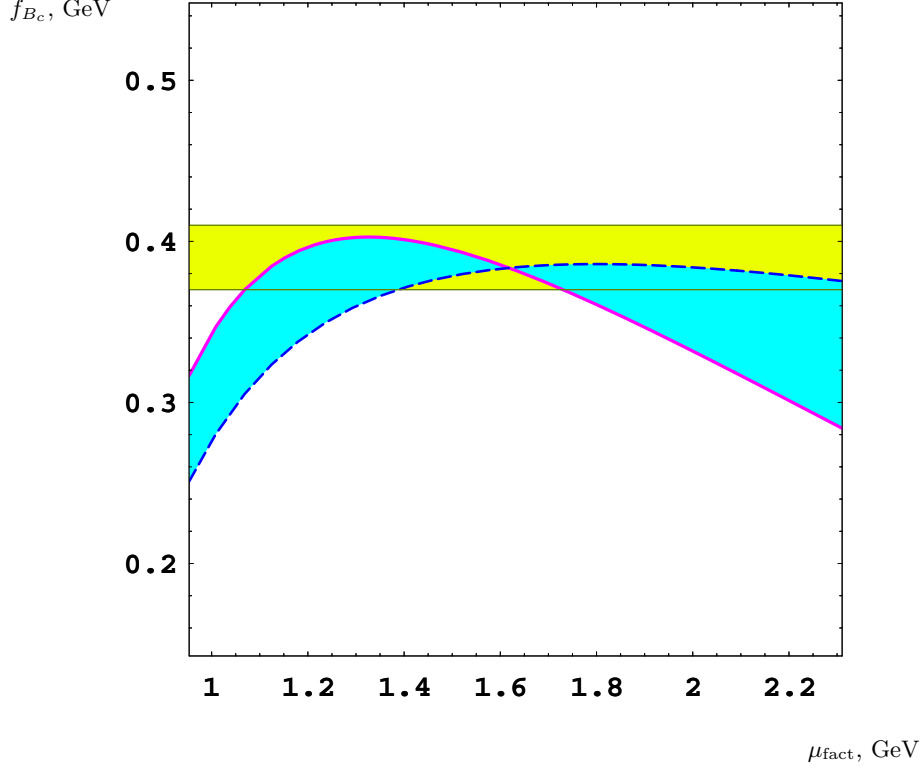


FIG. 3: The leptonic constant of ground pseudoscalar state in the system of heavy quarkonium  $\bar{b}c$  is presented versus the soft scale of normalization. The shaded region restricted by curves corresponds to the change of hard scale from  $\mu_{\text{hard}} = 3$  GeV (the dashed curve) to  $\mu_{\text{hard}} = 2$  GeV (the solid curve) with the initial condition for the evolution of normalization factor  $\mathcal{A}(\mu_{\text{fact}})$  posed in the form of  $\mathcal{A}(1.2 \text{ GeV}) = 1$  and  $\mathcal{A}(1 \text{ GeV}) = 1$ , respectively, in the matrix element of current given in the nonrelativistic representation. The horizontal band is the limits expected from the QCD sum rules [43] and scaling relations for the leptonic constants of heavy quarkonia [44]. In the cross-point, the leptonic constant of  $B_c$  weakly depends on the parameters given by the hard scale of matching as well as the scale of the initial normalization.

The result of calculation for the leptonic constant of  $B_c$  in the potential approach is shown in Fig. 3. We chose the values of  $\mu_{\text{hard}}$  in a range, so that the stable value of leptonic constant would be posed at the scale  $\mu_{\text{fact}} < m_c^{\text{pole}}$ , which is the condition of consistency for the NRQCD approach. Then one can observe the position of  $\mu_{\text{fact}} \approx 1.5 - 1.6$  GeV, where the estimate of  $f_{B_c}$  is independent of the choice of  $\mu_{\text{hard}}$  and  $\mu_0$ , and the maximal values at the different choices of  $\mu_{\text{hard}}$  are also close to each other.

The final result of two-loop calculations is

$$f_{B_c} = 395 \pm 15 \text{ MeV}. \quad (24)$$

It should be compared with the estimate of potential model itself without the matching

$$f_{B_c}^{\text{NRQCD}} = 493 \text{ MeV}, \quad (25)$$

which indicates the magnitude of the correction about 20%. Furthermore, the calculations in the same potential model with the one-loop matching [40] gave

$$f_{B_c}^{1\text{-loop}} = 400 \pm 45 \text{ MeV}, \quad (26)$$

where the uncertainty is significantly greater than in the two-loop procedure, since at the one-loop level we have no stable point in the scale dependence of the result. Therefore, in contrast to the discussion given in [39] we see that the correction is not crucially large, but it is under control in the system of  $B_c$ . The reason for such the claim on the reliability of result is caused by two circumstances. First, the one-loop anomalous dimension of NRQCD current is equal to zero. Therefore, we start the summation of large logs in the framework of renormalization group (RG) with the expressions in (22) and (23). Second, after such the summation of large logs the three-loop corrections could be considered as small beyond the leading RG logs.

The result on  $f_{B_c}$  is in agreement with the scaling relation derived from the quasi-local QCD sum rules [44], which use the regularity in the heavy quarkonium mass spectra, i.e. the fact that the splitting between the quarkonium levels after the averaging over the spins of heavy quarks weakly depends on the quark flavors. So, the scaling law for the S-wave quarkonia has the form

$$\frac{f_n^2}{M_n} \left( \frac{M_n}{M_1} \right)^2 \left( \frac{m_1 + m_2}{4\mu_{12}} \right)^2 = \frac{c}{n}, \quad (27)$$

where  $n$  is the radial quantum number,  $m_{1,2}$  are the masses of heavy quarks composing the quarkonium,  $\mu_{12}$  is the reduced mass of quarks, and  $c$  is a dimensional constant independent of both the quark flavors and the level number  $n$ . The value of  $c$  is determined by the splitting between the  $2S$  and  $1S$  levels, or the average kinetic energy of heavy quarks, which is independent of the quark flavors and  $n$  with the accuracy accepted. The accuracy depends on the heavy quark masses, and it is discussed in [44] in detail. The parameter  $c$  can be extracted from the known leptonic constants of  $\psi$  and  $\Upsilon$ , so that the scaling relation gives

$$f_{B_c^*} \approx 400 \text{ MeV}$$

for the vector state. The difference between the leptonic constants for the pseudoscalar and vector  $1S$ -states is caused by the spin-dependent corrections, which are small. Numerically, we get  $|f_{B_c^*} - f_{B_c}|/f_{B_c^*} < 3\%$ , hence, the estimates obtained from the potential model and the scaling relation is in a good agreement with each other.

### C. Non-leptonic modes

In comparison with the inclusive non-leptonic widths, which can be estimated in the framework of quark-hadron duality (see Table I), the calculations of exclusive modes usually involves the approximation of factorization [45], which, as expected, can be quite accurate for the  $B_c$ , since the quark-gluon sea is suppressed in the heavy quarkonium. Thus, the important parameters are the factors  $a_1$  and  $a_2$  in the non-leptonic weak lagrangian, which depend on the normalization point suitable for the  $B_c$  decays.

The agreement of QCD SR estimates for the non-leptonic decays of charmed quark in  $B_c$  with the values predicted by the potential models is rather good for the direct transitions with no permutation of color lines, i.e. the class I processes with the factor of  $a_1$  in the non-leptonic amplitude determined by the effective lagrangian. In contrast, the sum rule predictions are significantly enhanced in comparison with the values calculated in the potential models for the transitions with the color permutation, i.e. for the class II processes with the factor of  $a_2$  (see Table II).

Further, for the transitions, wherein the interference is significantly involved, the class III processes, we find that the absolute values of different terms given by the squares of  $a_1$  and  $a_2$  calculated in the sum rules are in agreement with the estimates of potential models. Taking into account the negative value of  $a_2$  with respect to  $a_1$ , we see the characteristic values of effects caused by the interference is about 35-50%.

At large recoils as in  $B_c^+ \rightarrow \psi\pi^+(\rho^+)$ , the spectator picture of transition can be broken by the hard gluon exchanges [46]. The spin effects in such decays were studied in [47]. However, we emphasize that the significant rates of  $B_c$  decays to the P- and D-wave charmonium states point out that the corrections in the second order of the heavy-quark velocity in the heavy quarkonia under study could be quite essential and they can suppress the corresponding decay rates, since the relative momentum of heavy quarks inside the quarkonium if different from zero should enhance the virtuality of gluon exchange, which suppresses the decay amplitudes.

For the widths of non-leptonic  $c$ -quark decays we have found that the sum rule estimates are greater than those of potential models<sup>6</sup>. In this respect we check that the QCD SR calculations are consistent with the inclusive ones. So, we sum up the calculated exclusive widths and estimate the total width of  $B_c$  meson as shown in Fig. 1, which points to a good agreement of our calculations with those of OPE and semi-inclusive estimates.

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<sup>6</sup> See also recent discussions of the  $B_c$  decays in [48, 49, 50, 51, 52, 53, 54].

Another interesting point is the possibility to extract the factorization parameters  $a_1$  and  $a_2$  in the  $c$ -quark decays by measuring the branching ratios

$$\frac{\Gamma[B_c^+ \rightarrow B^+ \bar{K}^0]}{\Gamma[B_c^+ \rightarrow B^0 K^+]} = \frac{\Gamma[B_c^+ \rightarrow B^+ \bar{K}^{*0}]}{\Gamma[B_c^+ \rightarrow B^0 K^{*+}]} = \frac{\Gamma[B_c^+ \rightarrow B^{*+} \bar{K}^0]}{\Gamma[B_c^+ \rightarrow B^{*0} K^+]} = \frac{\Gamma[B_c^+ \rightarrow B^{*+} \bar{K}^{*0}]}{\Gamma[B_c^+ \rightarrow B^{*0} K^{*+}]} = \frac{\Gamma_0}{\Gamma_+} = \left| \frac{V_{cs}}{V_{cd}} \right|^2 \left( \frac{a_2}{a_1} \right)^2. \quad (28)$$

This procedure can give the test for the factorization approach itself.

The suppressed decays caused by the flavor changing neutral currents were studied in [55].

The CP-violation in the  $B_c$  decays can be investigated in the same manner as made in the  $B$  decays. The expected CP-asymmetry of  $\mathcal{A}(B_c^\pm \rightarrow J/\psi D^\pm)$  is about  $4 \cdot 10^{-3}$ , when the corresponding branching ratio is suppressed as  $10^{-4}$  [2]. Thus, the direct study of CP-violation in the  $B_c$  decays is practically difficult because of low relative yield of  $B_c$  with respect to ordinary  $B$  mesons:  $\sigma(B_c)/\sigma(B) \sim 10^{-3}$ . A model-independent way to extract the CKM angle  $\gamma$  based on the measurement of two reference triangles was independently offered by Masetti, Fleischer and Wyler in [2] by investigating the modes with the neutral charmed meson in the final state (see the corresponding subsection below).

Another possibility is the lepton tagging of  $B_s$  in the  $B_c^\pm \rightarrow B_s^{(*)} l^\pm \nu$  decays for the study of mixing and CP-violation in the  $B_s$  sector [56].

### 1. CP-Violation in Decays of $B_c$ Meson

The  $B_c$  meson first observed by the CDF collaboration at FNAL [5] is expected to be copiously produced in the future experiments at hadron colliders [6] with facilities oriented to the study of fine effects in the heavy quark interactions such as the parameters of CP-violation and charged weak current mixing<sup>7</sup>. So, one could investigate the spectroscopy, production mechanism and decay features of  $B_c$  [6, 7, 8, 9, 10, 11, 14] with the incoming sample of several billion events. In such circumstances, in addition to the current success in the experimental study of decays with the CP-violation in the gold-plated mode of neutral  $B$ -meson by the BaBar and Belle collaborations [57] allowing one to extract the CKM-matrix angle  $\beta$  in the unitarity triangle, a possible challenge is whether one could get an opportunity to extract some information about the CKM unitarity triangle from the  $B_c$  physics in a model independent way or not. The theoretical principal answer is one can do it. Indeed, there is an intriguing opportunity to extract the angle  $\gamma$  in the model-independent way using the strategy of reference triangles [58] in the decays of doubly heavy hadrons. This ideology for the study of CP-violation in  $B_c$  decays was originally offered by M.Masetti [2], independently investigated by R.Fleischer and D.Wyler [2] and extended to the case of doubly heavy baryons<sup>8</sup> in [4].

Let us point out necessary conditions to extract the CP-violation effects in the model-independent way.

1. Interference. The measured quantities have to involve the amplitudes including both the CP-odd and CP-even phases.
2. Exclusive channels. The hadronic final state has to be fixed in order to isolate a definite flavor contents and, hence, the definite matrix elements of CKM matrix, which can exclude the interference of two CP-odd phases with indefinite CP-even phases due to strong interactions at both levels of the quark structure and the interactions in the final state.
3. Oscillations. The definite involvement of the CP-even phase is ensured by the oscillations taking place in the systems of neutral  $B$  or  $D$  mesons, wherein the CP-breaking effects can be systematically implemented.
4. Tagging. Once the oscillations are involved, the tagging of both the flavor and CP eigenstates is necessary for the complete procedure.

The gold-plated modes in the decays of neutral  $B$  mesons involve the oscillations of mesons themselves and, hence, they require the time-dependent measurements. In contrast, the decays of doubly heavy hadrons such as the  $B_c$  meson and  $\Xi_{bc}$  baryons with the neutral  $D^0$  or  $\bar{D}^0$  meson in the final state do not require the time-dependent measurements. The triangle ideology is based on the direct determination of absolute values for the set of four decays, at least: the

<sup>7</sup> See, for instance, the program on the B physics at Tevatron [12].

<sup>8</sup> A review on the physics of doubly heavy baryons is given in [1].

decays of hadron in the tagged  $D^0$  meson, the tagged  $\bar{D}^0$  meson, the tagged CP-even state<sup>9</sup> of  $D^0$ , and the decay of the anti-hadron into the tagged CP-even state of  $D^0$ . To illustrate, let us consider the decays of

$$B_c^+ \rightarrow D^0 D_s^+, \quad \text{and} \quad B_c^+ \rightarrow \bar{D}^0 D_s^+.$$

The corresponding diagrams with the decay of  $\bar{b}$ -quark are shown in Figs. 4 and 5. We stress that two diagrams of the decay to  $D^0$  have the additional negative sign caused by the Pauli interference of two charmed quarks, which, however, completely compensated after the Fierz transformation for the corresponding Dirac matrices.

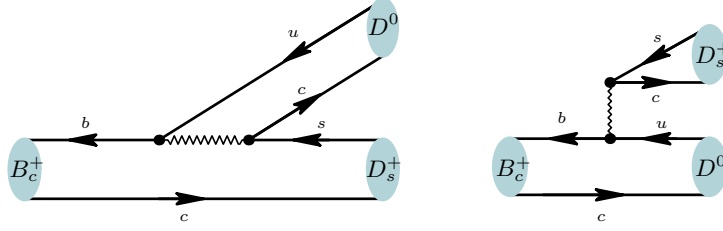


FIG. 4: The diagrams of  $\bar{b}$ -quark decay contributing to the weak transition  $B_c^+ \rightarrow D^0 D_s^+$ .

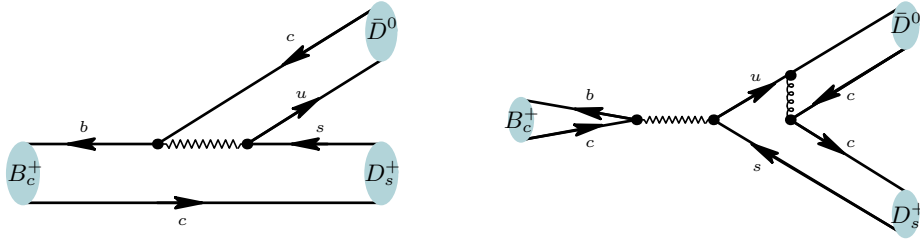


FIG. 5: The diagrams of  $\bar{b}$ -quark decay contributing to the weak transition  $B_c^+ \rightarrow \bar{D}^0 D_s^+$ .

The exclusive modes make the penguin terms to be excluded, since the penguins add an even number of charmed quarks, i.e. two or zero, while the final state contains two charmed quarks including one from the  $\bar{b}$  decay and one from the initial state. However, the diagram with the weak annihilation of two constituents, i.e. the charmed quark and beauty anti-quark in the  $B_c^+$  meson, can contribute in the next order in  $\alpha_s$  as shown in Fig. 5 for the given final state. Nevertheless, we see that such the diagrams have the same weak-interaction structure as at the tree level. Therefore, they do not break the consideration under interest. The magnitude of  $\alpha_s$ -correction to the absolute values of corresponding decay widths is discussed in [3].

Thus, the CP-odd phases of decays under consideration are determined by the tree-level diagrams shown in Figs. 4 and 5. Therefore, we can write down the amplitudes in the following form:

$$\mathcal{A}(B_c^+ \rightarrow D^0 D_s^+) \stackrel{\text{def}}{=} \mathcal{A}_D = V_{ub}^* V_{cs} \cdot \mathcal{M}_D, \quad \mathcal{A}(B_c^+ \rightarrow \bar{D}^0 D_s^+) \stackrel{\text{def}}{=} \mathcal{A}_{\bar{D}} = V_{cb}^* V_{us} \cdot \mathcal{M}_{\bar{D}}, \quad (29)$$

where  $\mathcal{M}_{\bar{D}, D}$  denote the CP-even factors depending on the dynamics of strong interactions. Using the definition of angle  $\gamma$

$$\gamma \stackrel{\text{def}}{=} -\arg \left[ \frac{V_{ub} V_{cs}^*}{V_{cb} V_{us}^*} \right],$$

for the CP-conjugated channels<sup>10</sup> we find

$$\mathcal{A}(B_c^- \rightarrow \bar{D}^0 D_s^-) = e^{-2i\gamma} \mathcal{A}_D, \quad \mathcal{A}(B_c^- \rightarrow D^0 D_s^-) = \mathcal{A}_{\bar{D}}. \quad (30)$$

<sup>9</sup> The CP-odd states of  $D^0$  can be used, too. However, their registration requires the detection of CP-even state of  $K^0$ , which can be complicated because of a detector construction, say, by a long base of  $K^0$  decay beyond a tracking system.

<sup>10</sup> For the sake of simplicity we put the overall phase of  $\arg V_{cb} V_{us}^* = 0$ , which corresponds to fixing the representation of the CKM matrix, e.g. by the Wolfenstein form [59].

We see that the corresponding widths for the decays to the flavor tagged modes coincide with the CP-conjugated ones. However, the story can be continued by using the definition of CP-eigenstates for the oscillating  $D^0 \leftrightarrow \bar{D}^0$  system<sup>11</sup>,

$$D_{1,2} = \frac{1}{\sqrt{2}}(D^0 \pm \bar{D}^0),$$

so that we straightforwardly get

$$\sqrt{2}\mathcal{A}(B_c^+ \rightarrow D_s^+ D_1) \stackrel{\text{def}}{=} \sqrt{2}\mathcal{A}_{D_1} = \mathcal{A}_D + \mathcal{A}_{\bar{D}}, \quad (31)$$

$$\sqrt{2}\mathcal{A}(B_c^- \rightarrow D_s^- D_1) \stackrel{\text{def}}{=} \sqrt{2}\mathcal{A}_{D_1}^{\text{CP}} = e^{-2i\gamma}\mathcal{A}_D + \mathcal{A}_{\bar{D}}. \quad (32)$$

The complex numbers entering (31) and (32) establish two triangles with the definite angle  $2\gamma$  between the vertex positions as shown in Fig. 6. Thus, due to the unitarity, the measurement of four absolute values

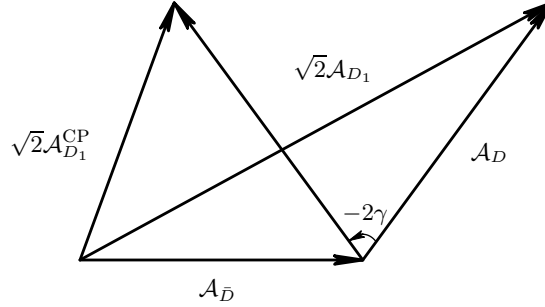


FIG. 6: The reference-triangles.

$$\begin{aligned} |\mathcal{A}_D| &= |\mathcal{A}(B_c^+ \rightarrow D_s^+ D^0)|, & |\mathcal{A}_{\bar{D}}| &= |\mathcal{A}(B_c^+ \rightarrow D_s^+ \bar{D}^0)|, \\ |\mathcal{A}_{D_1}| &= |\mathcal{A}(B_c^+ \rightarrow D_s^+ D_1)|, & |\mathcal{A}_{D_1}^{\text{CP}}| &= |\mathcal{A}(B_c^- \rightarrow D_s^- D_1)|, \end{aligned} \quad (33)$$

can constructively reproduce the angle  $\gamma$  in the model-independent way.

The above triangle-ideology can be implemented for the analogous decays to the excited states of charmed mesons in the final state.

The residual theoretical challenge is to evaluate the characteristic widths or branching fractions. We address this problem and analyze the color structure of amplitudes. So, we find that the matrix elements under interest have the different magnitudes of color suppression, so that at the tree level we get  $\mathcal{A}_D \sim O(\sqrt{N_c})$  and  $\mathcal{A}_{\bar{D}} \sim O(1/\sqrt{N_c})$ , while the ratio of relevant CKM-matrix elements,

$$\left| \frac{V_{ub}V_{cs}^*}{V_{cb}V_{us}^*} \right| \sim O(1)$$

with respect to the small parameter of Cabibbo angle,  $\lambda = \sin\theta_C$ , which one can easily find in the Wolfenstein parametrization

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}.$$

<sup>11</sup> The suppressed effects of CP-violation in the oscillations of neutral  $D$  mesons are irrelevant here, and we can neglect them in the sound way.

Nevertheless, the interference of two diagrams in the decays of  $B_c^+$  to the  $D^0$  meson is destructive, and the absolute values of the amplitudes  $\mathcal{A}_D$  and  $\mathcal{A}_{\bar{D}}$  become close to each other. Thus, we expect that the sides of the reference-triangles are of the same order of magnitude, which makes the method to be an attractive way to extract the angle  $\gamma$ .

The predictions of QCD sum rules for the exclusive decays of  $B_c$  are summarized in Table III at the fixed values of factors  $a_{1,2}$  and lifetime. For the sake of completeness and comparison we show the estimates for the channels with the neutral  $D$  meson and charged one  $D^+$  as well as for the vector states in addition to the pseudoscalar ones.

First, we see that the similar decay modes without the strange quark in the final state can be, in principle, used for the same extraction of CKM angle  $\gamma$ , however, this channels are more problematic from the methodic point of view, since the sides of reference-triangles significantly differ from each other<sup>12</sup>, so that the measurements have to be extremely accurate in order to get valuable information on the angle. Indeed, we should accumulate a huge statistics for the dominant mode in order to draw any conclusion on the consistency of triangle with a small side.

Second, the decay modes with the vector neutral  $D$  meson in the final state are useless for the purpose of the CKM measurement under the approach discussed. However, the modes with the vector charged  $D^*$  and  $D_s^*$  mesons can be important for the procedure of  $\gamma$  extraction. This note could be essential for the mode with  $D^{*+} \rightarrow D^0 \pi^+$  and  $D^0 \rightarrow K^- \pi^+$ , but, in this case, the presence of neutral charmed meson should be carefully treated in order to avoid the misidentification with the primary neutral charmed meson. In other case, we should use the mode with the neutral pion  $D^{*+} \rightarrow D^+ \pi^0$ , which detection in an experimental facility could be problematic. The same note is applicable for the vector  $D_s^{*+}$  meson, which radiative electromagnetic decay is problematic for the detection, too, since the photon could be loosed. However, the lose of the photon for the fully reconstructed  $D_s^+$  and  $B_c^+$  does not disturb the analysis.

TABLE III: Branching ratios of exclusive  $B_c^+$  decays at the fixed choice of factors:  $a_1^b = 1.14$  and  $a_2^b = -0.20$  in the non-leptonic decays of  $\bar{b}$  quark. The lifetime of  $B_c$  is appropriately normalized by  $\tau[B_c] \approx 0.45$  ps.

Mode	BR, $10^{-6}$	Mode	BR, $10^{-6}$
$B_c^+ \rightarrow D^+ \bar{D}^0$	53 [18]	$B_c^+ \rightarrow D^+ D^0$	0.32 [0.1]
$B_c^+ \rightarrow D^+ \bar{D}^{*0}$	75 [20]	$B_c^+ \rightarrow D^+ D^{*0}$	0.28 [0.07]
$B_c^+ \rightarrow D^{*+} \bar{D}^0$	49 [9]	$B_c^+ \rightarrow D^{*+} D^0$	0.40 [0.4]
$B_c^+ \rightarrow D^{*+} \bar{D}^{*0}$	330 [120]	$B_c^+ \rightarrow D^{*+} D^{*0}$	1.59 [0.4]
$B_c^+ \rightarrow D_s^+ \bar{D}^0$	4.8 [1]	$B_c^+ \rightarrow D_s^+ D^0$	6.6 [1.7]
$B_c^+ \rightarrow D_s^+ \bar{D}^{*0}$	7.1 [1.2]	$B_c^+ \rightarrow D_s^+ D^{*0}$	6.3 [1.3]
$B_c^+ \rightarrow D_s^{*+} \bar{D}^0$	4.5 [0.5]	$B_c^+ \rightarrow D_s^{*+} D^0$	8.5 [8.1]
$B_c^+ \rightarrow D_s^{*+} \bar{D}^{*0}$	26 [2]	$B_c^+ \rightarrow D_s^{*+} D^{*0}$	40.4 [6.2]

In the BTeV [12] and LHCb [13] experiments one expects the  $B_c$  production at the level of several billion events. Therefore, we predict  $10^4 - 10^5$  decays of  $B_c$  in the gold-plated modes under interest. The experimental challenge is the efficiency of detection. One usually get a 10%-efficiency for the observation of distinct secondary vertices outstanding from the primary vertex of beam interaction. Next, we have to take into account the branching ratios of  $D_s$  and  $D^0$  mesons. This efficiency crucially depends on whether we can detect the neutral kaons and pions or not. So, for the  $D_s$  meson the corresponding branching ratios grow from 4% (no neutral  $K$  and  $\pi$ ) to 25%. The same interval for the neutral  $D^0$  is from 11 to 31%. The detection of neutral kaon is necessary for the measurement of decay modes into the CP-odd state  $D_2$  of the neutral  $D^0$  meson, however, we can omit this cross-check channel from the analysis dealing with the CP-even state of  $D_1$ . The corresponding intervals of branching ratios reachable by the experiment are from 0.5 to 1.3% for the CP-even state and from 1.5 to 3.8% for the CP-odd state of  $D^0$ . The pessimistic estimate for the product of branching ratios is about  $2 \cdot 10^{-4}$ , which results in 2 – 20 reconstructed events. Thus, an acceptance of experimental facility and an opportunity to detect neutral pions and kaons as well as reliable estimates of total cross section for the  $B_c$  production in hadronic collisions are of importance in order to make expectations more accurate.

#### IV. CONCLUSION

We have reviewed the current status of theoretical predictions for the decays of  $B_c$  meson.

<sup>12</sup> The ratio of widths is basically determined by the factor of  $|V_{cb}V_{ud}a_2|^2/|V_{ub}V_{cd}a_1|^2 \sim 110$ , if we ignore the interference effects.

We have found that the various approaches: OPE, Potential models and QCD sum rules, result in the close estimates, while the SR as explored for the various heavy quark systems, lead to a smaller uncertainty due to quite an accurate knowledge of the heavy quark masses. So, summarizing we expect that the dominant contribution to the  $B_c$  lifetime is given by the charmed quark decays ( $\sim 70\%$ ), while the  $b$ -quark decays and the weak annihilation add about 20% and 10%, respectively. The predictions have been presented in the form of long tables with the numerical values of branching ratios. There are many physical points, which can be investigated in the  $B_c$  decays. We hope that this possibility could be, in various aspects, studied in the forthcoming experimental runs at hadron colliders, where the production rate about several billions  $B_c$  actually opens such the opportunity.

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